Settling the Size Matter

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Abstract

The size premium has failed to materialize since its discovery almost forty years ago, but is seemingly revived when controlling for quality-versus-junk exposures. This paper aims to resolve whether there exists a distinct size premium that can be captured in reality. For the US we confirm that a highly significant alpha emerges in regressions of size on quality, but for international markets we find that the size premium remains statistically indistinguishable from zero. Moreover, the US size premium appears to be beyond the practical reach of investors, because the alpha that is observed ex post in regressions cannot be captured by controlling for quality exposures ex ante. We also find that the significant regression alpha in the US is entirely driven by the short side of quality. Altogether, these results imply that size only adds value in conjunction with a short position in US junk stocks. However, we also show that small-cap exposure is vital for unlocking the full potential of other factors, such as value and momentum. We conclude that size is weak as a stand-alone factor but a powerful catalyst for other factors.

Keywords: size premium, quality, junk, profitability, factor investing, factor premiums, asset pricing, market efficiency, Fama-French model

JEL Classification: G11, G12, G14

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1. Introduction

The finding by Banz (1981) that small firms have higher risk-adjusted returns than large firms was one of the first major challenges to the Capital Asset Pricing Model (CAPM) and market efficiency in general. However, the size premium failed to materialize during the remainder of the 20th century, so it seemed that the size effect had disappeared after its discovery. During the first decade of the new millennium (2000-2009) small stocks made a strong come-back, and van Dijk (2011) cautioned that "the conclusion that the size effect has gone away is premature." This revival turned out to be short-lived though, as the size premium again failed to materialize in the subsequent decade (2010-2019); see, for example, Blitz (2020). Altogether, the size factor has a poor live track record since it was first published almost forty years ago.[1] Based on an extensive review of the size effect, Alquist, Israel, and Moskowitz (2018) conclude that there is "neither strong empirical evidence nor robust theoretical support for a prominent size premium." They do acknowledge that size can be an important factor for explaining mutual fund returns, and that other factors, such as value, tend to be more powerful among smaller stocks, which might be a reason to overweight small-cap stocks in long-only constrained portfolios. But "simply generically tilting toward small stocks is unlikely to provide much of a premium."

These findings contrast with those of Asness et al. (2018), who find that a strong and distinct size premium emerges from the same data if, crucially, one controls for the quality-versus-junk characteristics of firms. Their approach consists of time-series regressions of small-minus-big (SMB) portfolio returns on various control factors, including the quality-minus-junk (QMJ) factor of Asness, Frazzini, and Pedersen (2019). The SMB portfolio turns out to have a strong negative loading on the QMJ factor, indicating that the average small-cap stock has much poorer quality characteristics than the average large-cap stock. The alpha in the time-series regressions implies that if one accounts for the quality difference between small and large stocks, a highly significant and distinct small-cap premium is, in fact, clearly present in the US stock market. A similar result is found by Esakia et al. (2019) when controlling for the factors in the 5-factor model of Fama and French (2015). In particular, the size factor exhibits a large negative loading on the Fama-French profitability (RMW) factor, and the size premium also becomes significant when adjusting for that exposure. The similarity between the results of Asness et al. (2018) and Esakia (2019) is not surprising since the QMJ and RMW factors are highly correlated.[2] Hou and van Dijk (2019) also relate the significance of the size premium to interactions with profitability effects.

This paper examines the practical implications of these latest insights regarding the size premium. We confirm the result that a strong size premium emerges in time-series regressions of US small-minus-big portfolio returns on quality(-like) factors. For international stock markets we find that size also loads negatively on quality factors, but that the size premium remains

[1] It also appears that the size effect in the original study of Banz (1981) was overstated due to a delisting bias in returns that was only fixed after the publication. In this regard, Shumway (1997) documents that delistings for performance-related reasons are generally negative surprises. As delistings occur more frequently for smaller firms, the returns of small stocks were inflated in earlier studies. Consequently, using today's corrected data, the size premium has become substantially weaker over the in-sample period used in Banz (1981); see, e.g., Asness et al. (2018).

[2] The QMJ and RMW factors exhibit a correlation of 0.72 over the full sample (July 1963 to December 2019).

economically small and statistically indistinguishable from zero after controlling for these relations. Thus, the existence of a large and significant size premium is limited to the US market and does not carry over to international markets.

We next examine the size premium in the US more closely. The strong size premium that is observed in time-series regressions is an ex post result, and it is not obvious how this alpha could be isolated and captured with an investable strategy. We find that alternative SMB factors, which control for quality-versus-junk exposure ex ante, exhibit a similar performance as the standard SMB factor. Thus, the alpha that is observed in time-series regressions appears to be beyond the practical reach of investors. The regression results for the US market do imply that even a straightforward SMB portfolio is able to add significant value when combined with quality portfolios. Since factor investing strategies in practice typically follow a long-only approach, we further examine this result by making a breakdown of factors into their separate long and short legs, as in Blitz, Baltussen, and van Vliet (2020). Our tests show that the added value of SMB in time-series regressions is entirely driven by the short side of quality factors and that there is no size premium when controlling for the long side of quality factors. In other words, long-only factor strategies do not benefit from adding generic small-cap exposure.

In sum, the added value of size appears to be limited to investors who specifically short US junk stocks. However, this result does not imply that investors should generally strive for size neutrality, in particular when it comes to long-only factor strategies. As also argued by Alquist, Israel, and Moskowitz (2018), the fact that other factors, such as value, tend to be stronger in the small-cap space may justify a structural overweight in small-cap stocks even if the size premium itself is zero. We provide additional empirical support for this argument by showing that the Fama-French factors, which give a weight of 50% to the small-cap segment of the market, have highly significant alphas compared to the same factors without such a disproportionately high weight for small-caps. Thus, a tilt towards small-cap stocks in long-only factor strategies can serve as a powerful catalyst for unlocking the full potential of these other factors.

2. Data and methodology

Most of our data is obtained from publicly available sources, in particular the Kenneth French[3], AQR[4], and q-factor[5] data libraries. From the Kenneth French data library we gather returns for the market in excess of the risk-free rate (Mkt), and for the classic size (SMB), value (HML), and momentum (WML) factors. In addition, we obtain the recently introduced profitability (RMW) and investment (CMA) factors of Fama and French (2015). From the AQR data library we obtain the quality-minus-junk (QMJ) factor of Asness, Frazzini, and Pedersen (2019). From the q-library we obtain the investment (IA) and return on equity (ROE) factors from the Hou, Xue, and Zhang (2015) q-factor model, and the recently added expected growth (EG) factor described in Hou et al. (2020).

[3] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html
[4] https://www.aqr.com/Insights/Datasets
[5] http://global-q.org/index.html

The HML, WML, RMW, and CMA factors are all constructed using 2x3 independently sorted portfolios. On the size dimension, the universe is divided into big versus small stocks using the NYSE median market capitalization as breakpoint, and top, middle, and bottom portfolios on the various factor criteria are created using the NYSE 30th and 70th percentiles as breakpoints. HML uses the book-to-market ratio, WML the past 12-month return excluding the most recent month, RMW the operating profitability, and CMA the growth in total assets. The final factors are then calculated by taking a fifty-fifty long portfolio in the two capitalization-weighted top portfolios and a fifty-fifty short position in the two capitalization-weighted bottom portfolios. The QMJ factor is based on a mix of about twenty variables related to profitability, growth, safety, and payout, and is constructed using 2x3 conditional sorts (first on size, and then on quality) in Asness, Frazzini, and Pedersen (2019), and on 2x3 independent sorts in Asness et al. (2018). The QMJ series that is available online uses the former approach, i.e. conditional sorts. The IA and ROE factors are derived from 2x3x3 independent sorts, while the EG factor uses the standard 2x3 independent sorts.

The size factor, SMB, is constructed in a different fashion. A straightforward approach would be to simply take the difference between the value-weighted return of the stocks with above NYSE-median market capitalization versus the stocks with below NYSE-median market capitalization (SMB_{raw}). However, Fama and French (1993) argue that such an approach can bring along secondary exposures to value, since small stocks may have a different average book-to-market ratio than big stocks. In order to orthogonalize the SMB factor towards value, they compute it by taking the average of the three small-cap stock portfolios minus the average of the three big-cap stock portfolios from the same 2x3 sorts on size and value that are used to construct the HML value factor (SMB_{3F}). Similar to Asness et al. (2018) we will use this classic size factor as the base case for our analyses.

We also consider various alternative size factors, which all attempt to control for quality ex ante in one way or another. Our first alternative is the new size factor (SMB_{5F}) of Fama and French (2015), which replaces their classic size factor (SMB_{3F}). With the extension of the 3-factor model to a 5-factor model by adding profitability and investment factors, it would be somewhat arbitrary to orthogonalize the size factor to just the value factor, so Fama and French (2015) redefine the SMB factor by considering not only the 2x3 size/value sorts, but also the 2x3 size/profitability and 2x3 size/investment sorts, and taking the average of the 9 small-cap stock portfolios minus the average of the 9 big-cap stock portfolios. Since the loading on the profitability (RMW) factor is the main driver of the significant alpha for size in time-series regressions, we also consider a size factor which is solely based on the 2x3 size/profitability sorts (SMB_{RMW}). We also consider a size factor based on the 2x3 size/quality sorts (SMB_{QMJ}) available in the AQR data library, and a self-constructed size factor based on 2x3 independent sorts on size and the ex ante beta of a stock towards the QMJ factor (SMB_{beta_QMJ}).[6] Finally, we consider a size factor based on

[6] This beta is calculated by regressing the returns of a stock in excess of the risk-free return over the past 36 months on the market excess return, the 1-month lagged market excess return, the QMJ factor, and the 1-month lagged QMJ factor, and then taking the sum of the two estimated QMJ betas. For the construction of this factor (universe, breakpoints, weighting, etc.) we follow the methodology of Fama and French (1993, 2015). More specifically, we consider all common stocks (share codes 10 and 11) in the CRSP database traded on NYSE, NYSE MKT (formerly AMEX), and NASDAQ exchanges, calculate breakpoints on NYSE

the 2x3x3 independent sorts on size, investment (IA) and return on equity (ROE), by taking the average of the 9 small-cap stock portfolios minus the average of the 9 big-cap stock portfolios ($SMB_{IA\text{-}ROE}$), and a size factor based on the 2x3 independent sorts on size and expected growth (SMB_{EG}).

For the US, our sample covers the period from July 1963 to December 2019 (Fama-French/AQR data) or January 1967 to December 2019 (q-factor data), which are the longest periods for which all the required data series are available. We also consider international samples, with data from July 1990 to December 2019 (Fama-French data) or July 1993 to December 2019 (AQR data). All portfolios are capitalization-weighted, and all returns are in US dollars.

3. US versus international results

Time-series regression results for the US size factor (SMB_{3F}) are reported in Exhibit 1. The raw size premium amounts to 0.19% per month with a t-statistic of 1.68, which is weakly significant at the 10% confidence level. However, this size premium drops to an insignificant 0.08% after adjusting for market beta exposure. Additionally adjusting for lagged market beta exposure to account for non-synchronous trading of small stocks, as in Asness et al. (2018), the size premium drops further to a very meager 0.02%. The size premium remains absent when additionally controlling for the classic value and momentum factors, but jumps to 0.22% when the two new Fama-French factors are added. The associated t-statistic is 2.05, which is just above the standard 5% confidence level for statistical significance. The main driver of this boost is a highly significant negative loading on the profitability factor, RMW. This result is in line with Esakia et al. (2019).

INSERT EXHIBIT 1 ABOUT HERE

When instead of the two new Fama-French factors we plug in the QMJ factor of Asness, Frazzini, and Pedersen (2019), the size premium jumps to 0.42% per month, with a highly significant t-statistic of 3.98, driven by a strong negative loading on the QMJ factor. This finding is fully in line with the results of Asness et al. (2018), and confirms their conclusion that controlling for quality-versus-junk exposure reveals the presence of a strong and distinct size premium in the US stock market. Using the extended q-model instead of the Fama-French/AQR models we find an even stronger size premium, amounting to 0.60% per month with a t-statistic of 4.90. In this case the alpha is driven by highly significant negative loadings on the IA, ROE and EG factors. This appears to be yet another manifestation of the same phenomenon, since ROE and EG are also quality-related factors.[7]

We proceed by examining if the findings for the US carry over to international markets. Exhibits 2 and 3 show the results of similar time-series regressions for international regions, the former one containing results based on the Kenneth French data, and the latter one containing results that are fully based on AQR data. In all instances we observe that the raw size premium is essentially non-existent, or even has the wrong sign, and that this does not change after

stocks, and apply value-weighting within portfolios. Furthermore, we include CRSP delisting returns and we require 36 valid return observations for the beta estimation.

[7] The ROE and EG factors have full-sample correlations with the QMJ factor of 0.68 and 0.61 respectively.

controlling for market exposure and the traditional value and momentum factors. Controlling additionally for the two new Fama-French factors, profitability and investment, hardly changes the results. Using QMJ as a control factor instead, we find consistently negative loadings on this factor, many of which are highly significant, leading to higher estimated alphas. This result is again consistent with Asness et al. (2018). Crucially, however, we also find that the size premium remains statistically insignificant in all instances.[8]

INSERT EXHIBIT 2 ABOUT HERE

INSERT EXHIBIT 3 ABOUT HERE

Exhibit 4 provides additional international results at the country level, based on data from the AQR data library. We observe that the US is the only market for which the t-statistic of the size premium is above 2 after controlling for the QMJ factor. Among the other 23 countries there are only 3 countries which have a size premium that is weakly significant at the 10% level. For the other 20 countries the size premium remains economically weak and statistically insignificant. In sum, although controlling for quality versus junk exposure consistently increases the estimated magnitude of the size premium, the US is the sole market for which this effect is strong enough to obtain an indisputably significant size premium. For international markets the size premium generally remains statistically indistinguishable from zero, even after controlling for the significant quality-versus-junk exposures.

INSERT EXHIBIT 4 ABOUT HERE

4. Zooming in on the US evidence

The time-series evidence indicates that there exists a strong size premium in the US, provided that one controls for quality-versus-junk exposure. However, this is an ex post regression result, and it is not evident how this premium could be captured with an actual investment strategy. Controlling for quality-versus-junk exposure ex ante would seem to be the obvious way to go about this. This adjustment may be done in various ways. As described in the methodology section, we will consider the new size factor from the Fama-French 5-factor model (SMB_{5F}), a size factor based on 2x3 size/profitability sorts (SMB_{RMW}), a size factor based on 2x3 size/quality sorts (SMB_{QMJ}), a size factor based on 2x3 sorts on size and the past 36-month beta of a stock towards the QMJ factor (SMB_{beta_QMJ}), a size factor based on the 2x3x3 size/IA/ROE sorts ($SMB_{IA\text{-}ROE}$), and a size factor based on the 2x3 size/expected growth sorts (SMB_{EG}). Conditioning the size factor on quality factors by construction can be expected to result in a higher raw premium, and a neutral, or at least much lower, ex post loading on quality factors in time-series regressions.

INSERT EXHIBIT 5 ABOUT HERE

Exhibit 5 shows the results for the various quality-conditioned US size factors. We find that the alternative size factors have either similar or somewhat higher raw returns, but all of these returns

[8] This lack of significance cannot be attributed to the shorter sample period that is available for the international markets; if these same alphas had been observed over a much longer period (e.g. starting in 1963) they would remain insignificant.

fall well short of the alphas that were observed in Exhibit 1. Moreover, we find t-statistics varying between 0.17 and 1.19 when controlling for contemporaneous and lagged market exposure, so none of the alternative size factors even manages to come close to statistical significance after controlling for basic market exposure. Granted, the alpha is entirely back again in the regressions which control for quality exposure, but the point was to turn this alpha into actual return by neutralizing the size factor for such exposures. However, the quality-conditioned size factors all still exhibit huge quality exposures ex post. In sum, the size premium that is clearly visible ex post in time-series regressions appears to be beyond the reach of investors, as none of the investable strategies that aim to control for quality ex ante are able to extract this alpha.[9]

Although it is hard to isolate and capture the alpha of US size factor, the regression results do imply that SMB adds significant value when used in conjunction with quality factors. In other words, investors with quality exposure can expand the efficient frontier by adding size exposure to their portfolios. However, factor investing is often implemented with long-only (e.g. smart beta) portfolios, instead of long-short strategies such as the QMJ factor. Long-only investors looking to harvest the quality premium would go long quality stocks, but not short junk stocks. We therefore examine to which extent the added value of small-caps is driven by the long leg versus the short leg of quality factors. We do so by following the approach of Blitz, Baltussen, and van Vliet (2020), who break up each factor into their separate long and short legs. They do so using the same 2x3 sorts that are used to construct the standard factors. Where a standard long-short factor is constructed calculated by taking a fifty-fifty long portfolio in the two top portfolios and a fifty-fifty short position in the two bottom portfolios, the long leg is defined as a fifty-fifty long portfolio in the two top portfolios and a fifty-fifty short position in the two *middle* portfolios, and the short leg is defined as a fifty-fifty long portfolio in the two *middle* portfolios and a fifty-fifty short position in the two bottom portfolios. Note that the long leg and the short leg sum up to the original factor in this way.

INSERT EXHIBIT 6 ABOUT HERE

Exhibit 6 shows the results of regressing the SMB factor on either the long legs of the other factors or the short legs of the other factors. The main finding is that the alpha of SMB completely vanishes if we regress on just the long legs of other factors, and only remains in the regressions on the short legs of other factors. This result implies that there is no added value from SMB for long-only factor investors, and that only investors with positions in the short legs of quality factors may benefit from adding SMB exposure to their portfolio. The mechanism behind this is

[9] Asness et al. (2018) report a high raw return for a size factor that is ex ante conditioned on QMJ using 5x5 sorts. Since these 5x5 size/quality portfolios are not publicly available we cannot reproduce these results; however, we constructed a similar factor based on the 5x5 size/profitability portfolios in the Kenneth French data library. In unreported tests we observe that the raw return for this 5x5 profitability-conditioned size factor is very close to the raw return for the 5x5 quality-conditioned size factor in Asness et al. (2018), but, similar to our various 2x3 quality-conditioned size factors, the alpha is statistically insignificant after controlling for plain market exposure. In this paper we do not examine more granular size portfolios because of investability concerns. The small-cap legs of our various 2x3-based SMB factors comprise roughly 10% of total market capitalization. Switching to 5x5 portfolios means defining the small-cap portfolio as all stocks below the NYSE 20th percentile of market capitalization (instead of median level), so descending into the illiquid micro-cap space, which contains only about 3% of total market capitalization.

that the negative loadings of size on quality factors mainly stems from the short legs of those quality factors. Size has much smaller, or even insignificant loadings on the long legs of quality factors, and even highly positive loadings on the long legs of the investment factors (CMA and IA), so controlling for these exposures does not boost the size premium to significance. Altogether, the added value of size appears to be limited to investors in US equities who specifically short junk stocks.

5. Small-caps are a powerful catalyst

If there is no size premium, then one might be tempted to conclude that investors should strive for size neutral portfolios. However, this ignores the existence of powerful interaction effects between size and other factors, such as value. In particular, it is well known that factor premiums tend to be bigger among small-cap stocks than among large-cap stocks; see, for instance, Fama and French (2012, 2015). The Fama-French factor construction methodology takes advantage of this phenomenon by giving a disproportionately high weight to factor performance in the small-cap segment of the market. For instance, the classic HML value factor equally weights value-minus-growth long-short portfolios in the big-cap space and the small-cap space, even though big-cap stocks account for approximately 90% of total market capitalization. The academic long-short factors maintain a net zero exposure towards big-caps and small-caps, but long-only investors who want to benefit from the stronger performance of factors in the small-cap space have little choice but to accept an overweight position in small-cap stocks in their portfolios. The reason for this is that in long-only portfolios stocks cannot be underweighted by more than their benchmark weight, which is close to negligible for small stocks. Thus, overweight positions in small stocks that are in the long legs of factor portfolios cannot be offset with similar-sized underweight positions in small stocks that are in the short legs of factor portfolios. In the words of Alquist, Israel, and Moskowitz (2018): "Other factors working better in small caps can be a reason to overweight small stocks even though there is no pure size effect."

A proper evaluation of the attractiveness of factor premiums in the small-cap space versus the large-cap space should not be based on raw returns, but on risk-adjusted returns. For instance, if the value premium would be 50% higher among small-caps than among big-caps, but the risk involved would be 100% higher, then on a risk-adjusted basis the value premium would actually be weaker among small-caps. We formally examine the added value of giving more weight to the small-cap segment of the market when targeting other factors by regressing the standard academic factors on adjusted factors, which do not give a disproportionally large weight to small-cap stocks. We define the adjusted factors by using weights of 90% and 10% instead of 50% and 50%. For instance, the adjusted HML factor takes a 90% long-short position in big-cap value-versus-growth stocks and a 10% long-short position in small-cap value-versus-growth stocks, in accordance with the actual market capitalization of big and small stocks.

INSERT EXHIBIT 7 ABOUT HERE

The results are reported in Exhibit 7, where the adjusted factors are denoted with an asterisk (*). We find that the standard academic factors load heavily on their adjusted versions, but have consistently positive alphas. Moreover, these alphas are statistically significant in every instance, with just a single exception (RMW in the US). These results imply that giving a disproportionately high weight to the small-cap segment of the market when targeting factor premiums provides unique additional alpha, that cannot be obtained with factors in which the role of small-cap stocks is limited to their weight in the market portfolio. For long-only investors this means that an overweight in small-cap stocks may be desirable even if there is no size premium, because small-cap stocks can serve as a powerful catalyst for unlocking the full potential of other factors, such as value and momentum. The higher expected return from targeting other factors in the small-cap space has to be balanced against the systematic risk that comes along with small-cap exposure, in particular the risk of small-cap stocks in general lagging the capitalization-weighted index by a substantial amount or for a prolonged period of time.

6. Conclusions

The size premium has failed to materialize in the forty years after its discovery, but recent studies have shown that a highly significant size premium does emerge in time-series regressions which control for quality-versus-junk exposures. If there exists a strong and distinct size premium after all, then this has important theoretical asset pricing implications and also important practical investment implications. The objective of this paper has been to resolve the ambiguity surrounding the size premium.

We confirm the result that controlling for quality-versus-junk in time-series regressions leads to a significant size premium in the US stock market. For international markets, however, we find that although size also loads negatively on quality-versus-junk, this relation is not strong enough to obtain a statistically significant size premium, which makes the US the exception rather than the rule. Zooming in on the US results we find that multiple attempts at controlling for quality-versus-junk exposure ex ante fail to capture the size premium that shows up ex post in time-series regressions. Thus, it seems that this alpha is beyond the reach of investors. Although it is hard to isolate the alpha of the US size factor, the regression results do imply that size adds significant value when used in conjunction with quality. Taking a closer look at this result, we find that it is entirely driven by the short side of quality portfolios and breaks down for the long side. In sum, the added value of size appears to be limited to investors who short US junk stocks.

In the absence of a size premium one might be tempted to conclude that investors should strive for size neutral portfolios. However, this ignores the existence of powerful interaction effects between size and other factors, such as value. In particular, it is well known that factor premiums tend to be bigger among small-cap stocks than among large-cap stocks. We show that standard factor portfolios, which feature a disproportionately high weight for small-cap stocks, have large and highly significant alphas compared to factor portfolios without this feature. This means that size can add a lot of value by serving as a catalyst that helps to unlock the full potential of other factors, such as value and momentum. This interaction between size and other factors may already be a sufficient reason for long-only investors to systematically overweight small-cap stocks, regardless of whether the size characteristic itself is rewarded with a premium.

Exhibit 1: Regression results for US SMB_{3F} factor

	alpha	Mkt	Mkt(-1)	HML	WML	RMW	CMA	QMJ	IA	ROE	EG
coeff.	0.19										
t-stat.	(1.67)										
coeff.	0.08	0.20									
t-stat.	(0.75)	(8.01)									
coeff.	0.02	0.20	0.12								
t-stat.	(0.21)	(7.83)	(4.84)								
coeff.	0.06	0.18	0.13	-0.13	0.01						
t-stat.	(0.56)	(6.73)	(5.02)	(-3.25)	(0.53)						
coeff.	0.22	0.12	0.12	-0.07	0.04	-0.50	-0.11				
t-stat.	(2.05)	(4.38)	(5.24)	(-1.40)	(1.56)	(-10.10)	(-1.40)				
coeff.	0.42	-0.01	0.09	-0.21	0.08			-0.68			
t-stat.	(3.98)	(-0.24)	(4.19)	(-5.62)	(3.04)			(-12.65)			
coeff.	0.60	0.07	0.09						-0.21	-0.28	-0.35
t-stat.	(4.90)	(2.45)	(3.75)						(-3.28)	(-5.55)	(-4.69)

Sample period: July 1963 to December 2019 for all except the last regression, which is from January 1967 to December 2019.

Exhibit 2: Regression results for international SMB_{3F} factors, Kenneth French regional data

Europe	alpha	Mkt	Mkt(-1)	HML	WML	RMW	CMA
coeff.	-0.01						
t-stat.	(-0.08)						
coeff.	0.03	-0.07					
t-stat.	(0.23)	(-3.06)					
coeff.	-0.04	-0.09	0.15				
t-stat.	(-0.37)	(-3.87)	(6.87)				
coeff.	-0.04	-0.08	0.15	-0.05	0.01		
t-stat.	(-0.36)	(-3.34)	(6.91)	(-1.09)	(0.39)		
coeff.	0.02	-0.09	0.15	-0.07	0.03	-0.14	-0.05
t-stat.	(0.17)	(-3.44)	(6.69)	(-1.03)	(0.86)	(-1.56)	(-0.55)

Japan	alpha	Mkt	Mkt(-1)	HML	WML	RMW	CMA
coeff.	0.05						
t-stat.	(0.30)						
coeff.	0.04	0.08					
t-stat.	(0.26)	(2.57)					
coeff.	0.04	0.07	0.09				
t-stat.	(0.22)	(2.39)	(3.01)				
coeff.	0.05	0.06	0.09	-0.05	-0.04		
t-stat.	(0.32)	(1.99)	(2.94)	(-0.79)	(-0.99)		
coeff.	0.08	0.04	0.11	-0.21	-0.02	-0.02	0.35
t-stat.	(0.48)	(1.37)	(3.71)	(-2.91)	(-0.48)	(-0.17)	(3.48)

Asia-Pacific ex Japan	alpha	Mkt	Mkt(-1)	HML	WML	RMW	CMA
coeff.	-0.29						
t-stat.	(-1.85)						
coeff.	-0.28	-0.01					
t-stat.	(-1.79)	(-0.43)					
coeff.	-0.37	-0.03	0.16				
t-stat.	(-2.48)	(-1.14)	(6.04)				
coeff.	-0.39	-0.02	0.16	-0.06	0.05		
t-stat.	(-2.47)	(-0.70)	(6.28)	(-1.16)	(1.35)		
coeff.	-0.11	-0.09	0.15	-0.22	0.06	-0.36	-0.09
t-stat.	(-0.70)	(-2.89)	(5.99)	(-3.30)	(1.58)	(-4.76)	(-1.26)

Emerging Markets	alpha	Mkt	Mkt(-1)	HML	WML	RMW	CMA
coeff.	0.09						
t-stat.	(0.74)						
coeff.	0.09	-0.09					
t-stat.	(0.75)	(-4.80)					
coeff.	0.04	-0.11	0.11				
t-stat.	(0.32)	(-6.03)	(5.59)				
coeff.	-0.02	-0.10	0.11	-0.06	0.10		
t-stat.	(-0.12)	(-5.39)	(5.83)	(-1.10)	(2.48)		
coeff.	0.09	-0.12	0.10	-0.12	0.11	-0.22	-0.06
t-stat.	(0.70)	(-5.86)	(5.28)	(-2.02)	(2.76)	(-2.87)	(-0.92)

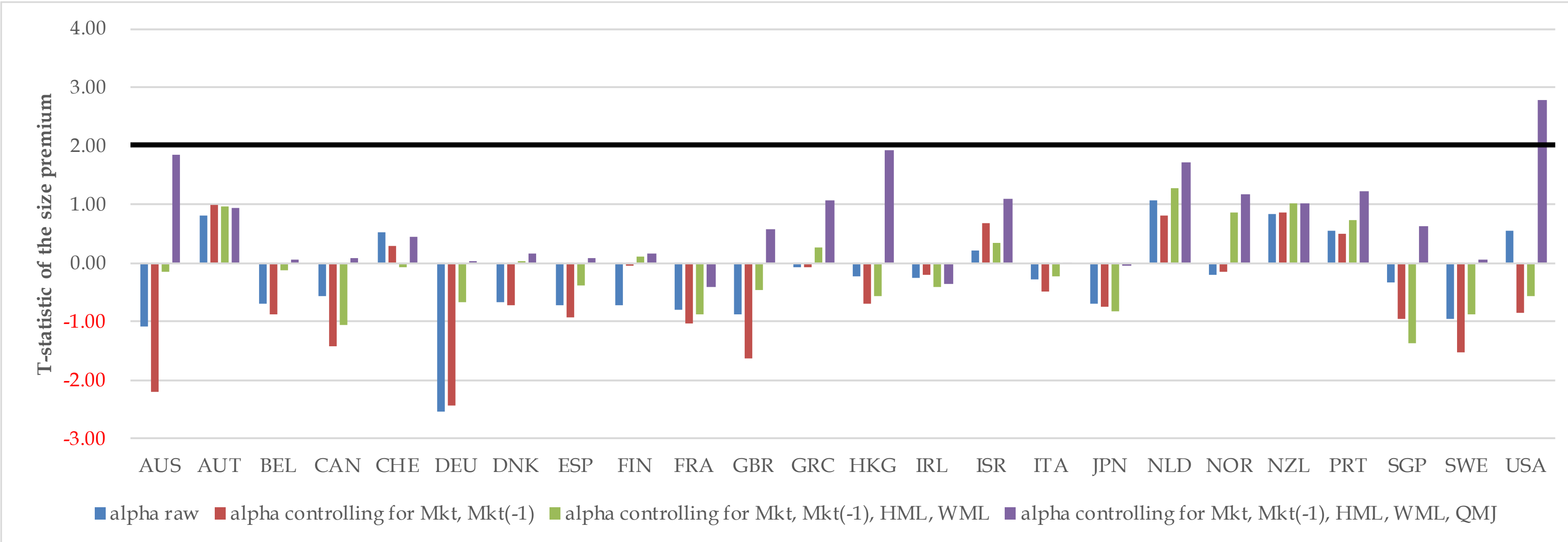
Sample period: July 1990 to December 2019 for all regions except Emerging Markets, which is from July 1991 to December 2019.

Exhibit 3: Regression results for international SMB_{3F} factors, AQR regional data

	alpha	Mkt	Mkt(-1)	HML	WML	QMJ
Europe						
coeff.	-0.12					
t-stat.	(-1.06)					
coeff.	-0.19	-0.03	0.17			
t-stat.	(-1.83)	(-1.54)	(7.94)			
coeff.	-0.11	-0.04	0.17	-0.10	-0.03	
t-stat.	(-1.05)	(-1.92)	(8.07)	(-2.26)	(-1.23)	
coeff.	0.00	-0.11	0.16	-0.14	0.03	-0.27
t-stat.	(-0.03)	(-3.82)	(7.54)	(-3.03)	(1.01)	(-3.60)
Pacific						
coeff.	-0.16					
t-stat.	(-1.40)					
coeff.	-0.18	0.02	0.10			
t-stat.	(-1.68)	(0.97)	(4.59)			
coeff.	-0.20	0.02	0.10	0.05	-0.01	
t-stat.	(-1.75)	(0.93)	(4.56)	(0.88)	(-0.37)	
coeff.	0.06	-0.12	0.10	-0.12	0.06	-0.46
t-stat.	(0.56)	(-4.28)	(4.86)	(-2.28)	(1.96)	(-7.72)

Sample period: July 1993 to December 2019.

Exhibit 4: Regression results for international SMB$_{3F}$ factors, AQR country data



Sample period: July 1993 (or later for some countries) to December 2019. The fat line denotes the threshold for statistical significance at the 5% confidence level.

Exhibit 5: Regression results for different versions of the US SMB factor

SMB$_{5F}$

	alpha	Mkt	Mkt(-1)	HML	WML	RMW	CMA	QMJ
coeff.	0.23							
t-stat.	(1.97)							
coeff.	0.13	0.19						
t-stat.	(1.12)	(7.49)						
coeff.	0.06	0.18	0.13					
t-stat.	(0.54)	(7.31)	(5.22)					
coeff.	0.05	0.18	0.13	0.00	0.01			
t-stat.	(0.43)	(7.00)	(5.23)	(0.07)	(0.46)			
coeff.	0.19	0.13	0.13	0.07	0.04	-0.43	-0.13	
t-stat.	(1.75)	(4.72)	(5.38)	(1.39)	(1.38)	(-8.59)	(-1.71)	
coeff.	0.39	0.01	0.10	-0.07	0.07			-0.65
t-stat.	(3.64)	(0.35)	(4.44)	(-1.86)	(2.82)			(-11.88)

SMB$_{RMW}$

	alpha	Mkt	Mkt(-1)	HML	WML	RMW	CMA	QMJ
coeff.	0.27							
t-stat.	(2.37)							
coeff.	0.18	0.17						
t-stat.	(1.60)	(6.79)						
coeff.	0.11	0.16	0.14					
t-stat.	(1.00)	(6.58)	(5.46)					
coeff.	0.07	0.18	0.13	0.08	0.02			
t-stat.	(0.59)	(6.84)	(5.42)	(1.97)	(0.60)			
coeff.	0.15	0.14	0.13	0.14	0.03	-0.24	-0.11	
t-stat.	(1.35)	(5.17)	(5.43)	(2.51)	(1.14)	(-4.67)	(-1.39)	
coeff.	0.33	0.04	0.11	0.02	0.06			-0.50
t-stat.	(2.99)	(1.50)	(4.73)	(0.61)	(2.36)			(-8.90)

SMB$_{QMJ}$

	alpha	Mkt	Mkt(-1)	HML	WML	RMW	CMA	QMJ
coeff.	0.20							
t-stat.	(1.78)							
coeff.	0.10	0.19						
t-stat.	(0.88)	(7.86)						
coeff.	0.02	0.18	0.15					
t-stat.	(0.17)	(7.69)	(6.44)					
coeff.	0.08	0.18	0.15	0.03	-0.09			
t-stat.	(0.69)	(7.07)	(6.20)	(0.70)	(-3.46)			
coeff.	0.21	0.12	0.15	0.11	-0.07	-0.38	-0.16	
t-stat.	(1.95)	(4.78)	(6.35)	(2.12)	(-2.70)	(-7.73)	(-2.11)	
coeff.	0.40	0.01	0.12	-0.04	-0.03			-0.61
t-stat.	(3.85)	(0.50)	(5.50)	(-1.14)	(-1.45)			(-11.65)

SMB$_{beta_QMJ}$

	alpha	Mkt	Mkt(-1)	HML	WML	RMW	CMA	QMJ
coeff.	0.27							
t-stat.	(2.42)							
coeff.	0.19	0.15						
t-stat.	(1.74)	(5.89)						
coeff.	0.12	0.14	0.14					
t-stat.	(1.12)	(5.67)	(5.66)					
coeff.	0.14	0.15	0.13	0.13	-0.09			
t-stat.	(1.24)	(5.91)	(5.40)	(3.42)	(-3.36)			
coeff.	0.23	0.11	0.13	0.17	-0.07	-0.31	-0.06	
t-stat.	(2.16)	(4.21)	(5.46)	(3.23)	(-2.80)	(-6.31)	(-0.78)	
coeff.	0.40	0.01	0.11	0.08	-0.04			-0.50
t-stat.	(3.76)	(0.48)	(4.69)	(2.07)	(-1.69)			(-9.30)

	alpha	Mkt	Mkt(-1)	IA	ROE	EG
$SMB_{IA\text{-}ROE}$						
coeff.	0.27					
t-stat.	(2.27)					
coeff.	0.17	0.19				
t-stat.	(1.49)	(7.16)				
coeff.	0.12	0.18	0.11			
t-stat.	(1.02)	(6.96)	(4.40)			
coeff.	0.56	0.09	0.09	-0.02	-0.20	-0.32
t-stat.	(4.34)	(3.17)	(3.52)	(-0.27)	(-3.95)	(-4.17)

	alpha	Mkt	Mkt(-1)	IA	ROE	EG
SMB_{EG}						
coeff.	0.34					
t-stat.	(2.63)					
coeff.	0.22	0.23				
t-stat.	(1.77)	(8.23)				
coeff.	0.15	0.22	0.15			
t-stat.	(1.19)	(8.04)	(5.63)			
coeff.	0.55	0.15	0.12	0.12	-0.40	-0.22
t-stat.	(4.18)	(5.13)	(4.69)	(1.83)	(-7.55)	(-2.76)

Sample period: July 1963 to December 2019 for SMB_{5F}, SMB_{RMW}, SMB_{QMJ}, and SMB_{beta_QMJ}, and January 1967 to December 2019 for $SMB_{IA\text{-}ROE}$ and SMB_{EG}.

Exhibit 6: Regression results for US SMB_{3F} on either the long legs or the short legs of other factors

	alpha	Mkt	Mkt(-1)	HML_L	WML_L	RMW_L	CMA_L	QMJ_L	IA_L	ROE_L	EG_L
coeff.	-0.10	0.08	0.09	0.20	0.32	0.19	0.77				
t-stat.	(-0.99)	(3.23)	(3.93)	(2.74)	(7.24)	(1.94)	(8.42)				
coeff.	-0.09	0.13	0.12	0.29	0.41			-0.22			
t-stat.	(-0.82)	(5.09)	(5.05)	(3.86)	(9.01)			(-2.11)			
coeff.	-0.07	0.17	0.11						0.50	0.00	0.18
t-stat.	(-0.59)	(6.25)	(4.15)						(4.71)	(0.02)	(1.74)

	alpha	Mkt	Mkt(-1)	HML_S	WML_S	RMW_S	CMA_S	QMJ_S	IA_S	ROE_S	EG_S
coeff.	0.31	0.01	0.09	0.03	-0.04	-0.64	-0.29				
t-stat.	(3.09)	(0.26)	(3.99)	(0.45)	(-1.04)	(-9.52)	(-2.96)				
coeff.	0.40	0.00	0.08	-0.19	0.00			-0.79			
t-stat.	(3.89)	(-0.01)	(3.50)	(-3.70)	(0.01)			(-10.31)			
coeff.	0.51	-0.01	0.08						-0.26	-0.35	-0.43
t-stat.	(4.69)	(-0.23)	(3.44)						(-3.24)	(-5.76)	(-4.80)

Sample period: July 1963 to December 2019 for the first two regressions, and January 1967 to December 2019 for the third regression.

Exhibit 7: Regression results for standard factors on alternative factors with 10% instead of 50% weight to small-caps

US, Fama-French factors

		alpha	Mkt-RF	SMB	HML*	WML*	RMW*	CMA*
HML	coeff.	0.14	-0.05	-0.08	**0.84**	0.00	0.13	0.10
	t-stat.	(4.22)	(-6.33)	(-6.85)	**(59.87)**	(-0.24)	(8.23)	(6.01)
WML	coeff.	0.23	0.00	-0.01	-0.04	**0.89**	0.01	0.05
	t-stat.	(5.79)	(-0.47)	(-1.05)	(-2.47)	**(98.84)**	(0.44)	(2.33)
RMW	coeff.	0.05	0.02	-0.08	0.09	-0.03	**0.85**	0.03
	t-stat.	(1.56)	(2.65)	(-6.39)	(5.81)	(-4.19)	**(52.52)**	(1.50)
CMA	coeff.	0.16	-0.04	0.01	0.05	0.02	-0.05	**0.68**
	t-stat.	(5.74)	(-5.89)	(0.62)	(3.86)	(2.41)	(-4.07)	**(47.81)**

Developed ex US, Fama-French factors

		alpha	Mkt-RF	SMB	HML*	WML*	RMW*	CMA*
HML	coeff.	0.13	-0.05	-0.04	**0.77**	0.00	0.10	0.18
	t-stat.	(3.28)	(-5.24)	(-1.91)	**(41.41)**	(-0.40)	(3.63)	(8.04)
WML	coeff.	0.29	-0.01	-0.03	-0.04	**0.85**	0.00	0.04
	t-stat.	(5.90)	(-1.07)	(-1.20)	(-1.71)	**(61.78)**	(0.13)	(1.66)
RMW	coeff.	0.17	-0.01	-0.02	0.04	-0.01	**0.67**	-0.06
	t-stat.	(5.86)	(-1.68)	(-1.47)	(3.08)	(-0.87)	**(34.32)**	(-3.55)
CMA	coeff.	0.07	-0.03	-0.05	0.06	0.01	0.02	**0.78**
	t-stat.	(2.15)	(-4.41)	(-3.12)	(4.03)	(1.31)	(0.94)	**(45.47)**

US, QMJ instead of RMW and CMA

		alpha	Mkt-RF	SMB	HML*	WML*	QMJ*
HML	coeff.	0.13	-0.05	-0.08	**0.91**	0.00	0.14
	t-stat.	(3.81)	(-5.89)	(-6.78)	**(66.89)**	(0.06)	(7.12)
WML	coeff.	0.23	-0.01	-0.02	-0.01	**0.89**	0.02
	t-stat.	(5.65)	(-0.76)	(-1.06)	(-0.77)	**(98.52)**	(1.03)
QMJ	coeff.	0.17	-0.06	-0.08	0.10	0.01	**0.84**
	t-stat.	(4.97)	(-6.88)	(-7.00)	(7.42)	(1.48)	**(43.04)**

US, q-factors

		alpha	Mkt-RF	SMB	IA*	ROE*	EG*
IA	coeff.	0.20	-0.03	0.01	**0.67**	-0.06	0.04
	t-stat.	(6.67)	(-4.84)	(1.01)	**(57.15)**	(-5.22)	(2.53)
ROE	coeff.	0.31	0.00	-0.05	0.07	**0.86**	-0.11
	t-stat.	(7.98)	(-0.19)	(-3.65)	(4.59)	**(57.23)**	(-5.94)
EG	coeff.	0.39	-0.02	-0.02	0.10	0.09	**0.63**
	t-stat.	(12.07)	(-2.87)	(-2.20)	(7.51)	(6.91)	**(40.60)**

Sample period: July 1963 to December 2019 for the US Fama-French and QMJ results, January 1967 to December 2019 for the US q-factor results, and July 1990 to December 2019 for the Developed ex US results.

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